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Assessing the Wave Propagation Velocity and Damping at Commodus Passage of Colosseum Using Microtremor Applying the CERS methods

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Abstract

The foundation of Colosseum is 12 m-thick spread foundation divided into upper and lower 6 m-thick layers. Commodus Passage was constructed with cutting the upper foundation about 60 m-long as a private passage to connect between the residence of emperor and Colosseum between 81 AD and 96 AD. Various investigations have been conducted for this passage to analyze directly the foundation structure. This paper describes the assessed wave propagation velocity and other characteristics applying the CERS <səːrz> methods with measuring microtremor on the floor of the passage. The CERS methods is an inclusive term of four methods as the C, E, R and S methods. The C or E methods are to realize realtime calculation of the wave propagation time and damping factor between a free end and a point in a media or reflecting plane, respectively. The R and S methods can calculate in realtime generally the time difference between two points separated spatially based on the basis of maximum cross-correlation as the R method or the basis of simple least error as the S method. The calculated result of the CERS methods can be checked mutually. Please see Nakamura [1] in detail. As a result, it can be estimated the wave propagation velocity of various wave motion component for every about 10 m on the floor. This result is agreeable with the P wave velocity structure by previous research.

Keywords: CERS methods, Foundation of Colosseum, Wave Propagation Velocity, Microtremor

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1. Introduction

The foundation of Colosseum built between 72 AD and 80 AD is 12 m-thick spread foundation divided into upper and lower 6 m-thick layers. Commodus Passage was constructed with cutting about 60 m-long as a private passage to connect between the residence of emperor and Colosseum during the reign of emperor Domitianus between 81 AD and 96 AD. Various investigations have been conducted for this passage because it is possible to analyze directly the foundation structure of Colosseum. This paper describes the estimated wave propagation velocity and other characteristics applying the CERS <sə:rz> methods with measuring microtremor on the floor of the passage. The CERS methods is an inclusive name of four methods, C, E, R and S methods, as Figure 1. The C method can calculate in realtime the wave propagation time and damping between a free end and a point among medium till the reflecting plane assuming a one dimensional wave field, and the E method can also calculate them between a free field and reflecting plane. Additionally, the R and S methods can generally calculate in realtime the time difference of waveforms between two points separated spatially based on the maximum cross-correlation basis and minimum simple error basis, respectively. The result of these four methods can be checked mutually. Please see Nakamura [1] for the detail of the CERS methods.

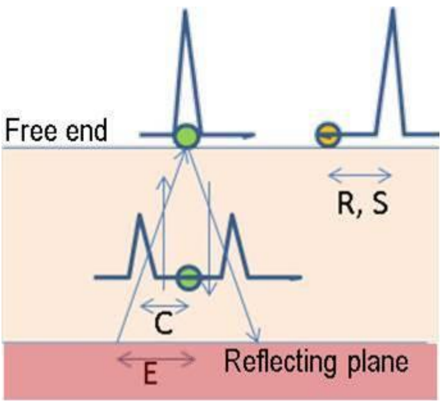


Figure 1. Concept of the CERS methods

2. Estimation method of the wave propagation velocity

(1) The R method

At first, the R method, estimating the wave propagation time between two points based on the maximum correlation function calculated from the microtremor measured at two points, is applied. Because it is considered that the source of microtremor is unclear and varies from hour to hour, the wave propagation time was presumed as the maximum value between estimated maximum time differences choosing as the large cross-correlation coefficient as possible in this instance. The detail is explaned in next section and also will be mentioned later.

(2) Concept of wave propagation time estimation applying the R or S methods for microtremor

Microtremor must be recorded for XYZ components setting the Y component as the direction of measurement line with sensors put on the measurement line. Figure 2 shows the image of the geological structure, sensor arrangement and measuring components.

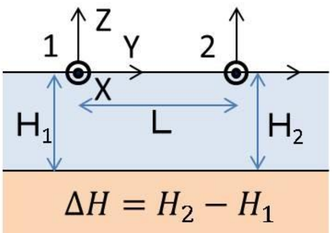


Figure 2. Image of sensor arragement

Recorded wave motion by the sensors at measurement points #1 and #2 is considered not only as direct delivered wave but also as a wave delivered as an refracted wave propagating at the lower layer. According to the wave direction and propagation direction, the sensor of the X direction is considered as measuring SH wave. In this case, although the both waves propagating at the upper and lower layers are SH wave, the wave propagating at the lower layer seems to become dominant because the lower layer has larger velocity and lower damping. In case of the Y or Z component, it is also possible to exist both a direct delivered wave and refracted wave, but the direct delivered wave must be dominant because the refracted wave accompanies with conversion of wave kind. In case of microtremor, wave motion comes from various directions. On the estimation of the wave propagation time, the maximum absolute time difference is considered with large cross correlation for the R method or small error for the S method.

Figure 3 shows the targeted wave motion of the direct delivered or refracted wave for each component, and the wave motion which tends to arise is marked pink. Here the wave propagation time is estimated as approximating the critical angle as vertical. And t_{*12} means the wave propagation from the measurement points #1 to #2.

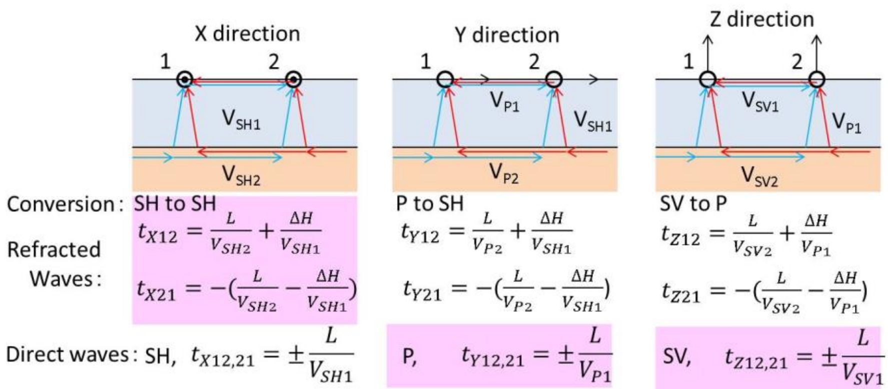


Figure 3. Variation of wave propagation between sensors #1 and #2

(3) The C and E methods

Because the wave motion propagating in the foundation horizontally is considered as reflecting multiply in the foundation, it is possible to set #1 as a measurement point at the free end and #2 as a measurement point in the medium with considering the wave motion field toward to the outer edge of the foundation from the arena. So the wave propagation time and the damping factor between these points are estimated with applying the C method. Then, if it is able to assume the foundation as the wave motion field with horizontal multiple reflection, it may be possible to estimate the wave propagation time and the damping factor of the foundation from the arena side to the outer edge applying the E method.

3. Microtremor measurement and result of its analysis
(1) Distribution of the measurement points

As seen in Figure 4, Commodus Passage of Colosseum passes linearly the foundation from the south side of the arena with cutting the upper foundation and then it is directly connected to the tunnel to east leaving from the foundation. Microtremor measurement was conducted at every about 10 m of the part along the outer side of the arena to the entrance of Commodus Passage with 10 m long, from the entrance to the terminal of the tunnel with 60 m long and from the edge of the foundation to the terminal of the connected tunnel with 10 m long. Microtremor was recorded simultaneously at two points among points #0 to #8, totally nine points. The measurement was conducted in the morning on Wednesday, 18th October, 2017 in local time and Figure 5 shows the distribution of the measured points with photographs during measurement. Figure 6 shows the location of measurement points from #1 to #7 in outline adding a vertical cross-section figure appeared on L. Orlando et al. [2]. The measurement instrument is NewPIC^{++} manufactured by System and

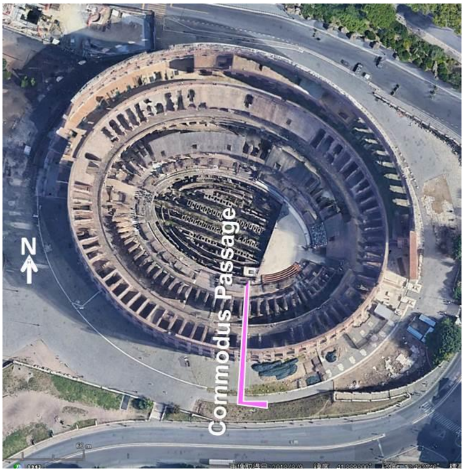


Figure 4. Commodus Passage of Colosseum

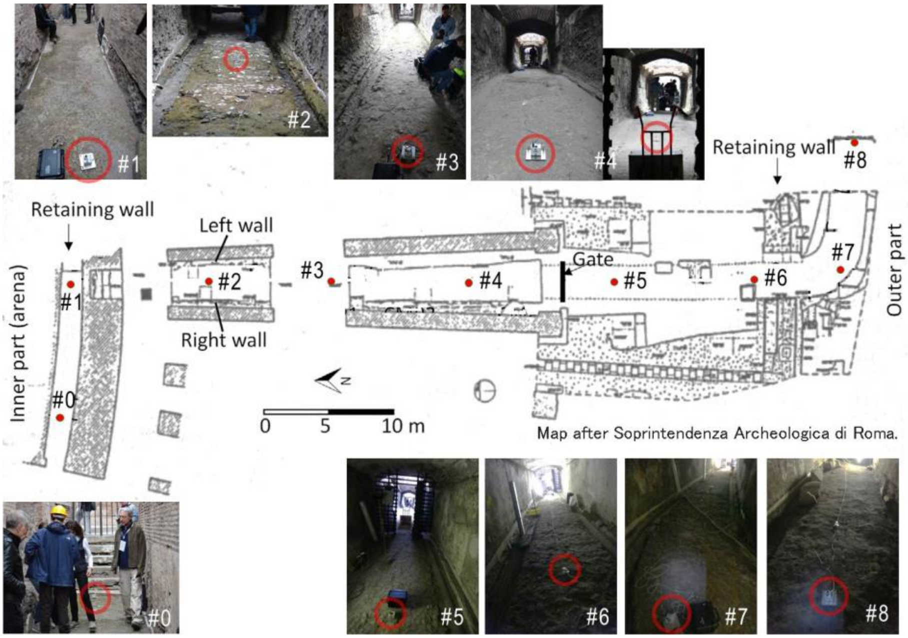


Figure 5. Measuring points for microtremor at Commodus Passage (Map adapted from L. Orlando et al. [2])

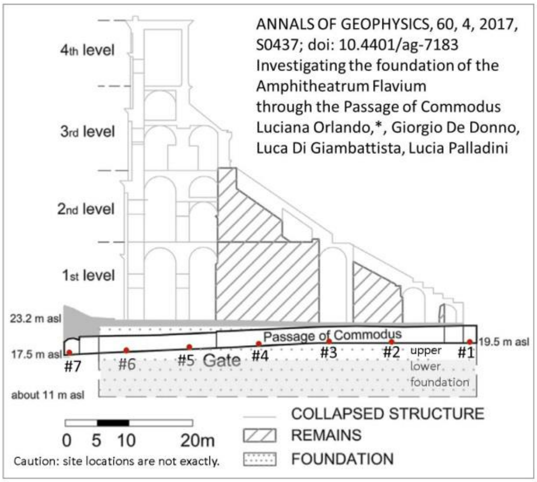


Figure 6. Measuring points at the passage (adapted from L. Orlando et al. [2])

Photo. 1. Instulment

Data Research, which can record accurate three-component waveform of two points simultaneously (see Photograph 1). Microtremor was recorded as eight sets of measurement point, #0 and #1, #2 and #1, #2 and #3, #4 and #3, #4 and #5, #6 and #5, #6 and #7, and #8 and #7, for five minutes waveform in 100 Hz sampling.

(2) The wave propagation time estimation applying the R method

Figure 7 represents an example for the time variation of time difference and cross-correlation coefficient between the points #2 and #1 applying the R method using exponential smoothing for averaging with half-life period of 10 seconds. As this figure, the time difference between these two points fluctuates widely corresponding to the various coming direction of the wave motion. The wave propagation time between the two points corresponds to the case when the coming direction of each wave motion coincides with the direction of measured line, and it corresponds to the maximum or minimum measured time difference. At this time, it becomes important to find the time difference with the maximum cross-correlation coefficient.

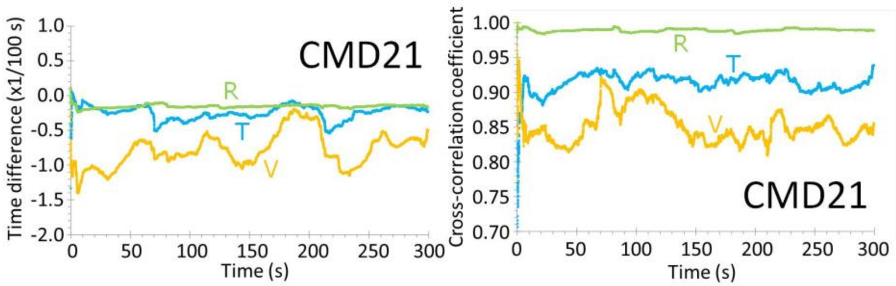


Figure 7. An example of measured time difference and cross-correlation coefficient between points #2 and #1 applying the R method

Figure 8 represents the relationship between the time difference and cross-correlation coefficient in case of the measurement set #2 and #1 when the horizontal axis is time difference and the vertical axis is the cross-correlation coefficient. This figure shows that the cross-correlation coefficient is basically over 0.6 for each direction as X(T), Y(R) and Z(V) without the tunnel on the foundation in the outer ground. Especially the coefficient becomes close to 1.0 for the R direction corresponding to the P wave. Also, this analysis used the calculation result of 290 seconds omitting the first and last 5 seconds data with consideration for existing of the transitional result at the first and last part of the cross-correlation calculation.

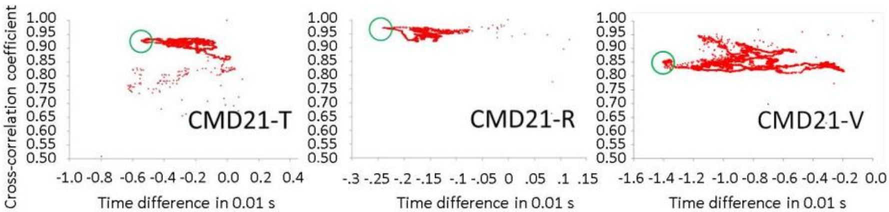


Figure 8. Examples of relationship between measured time difference and cross-correlation coefficient

(3) The wave propagation time estimation applying the C and E methods

It is possible to estimate the wave propagation time between the points #1 and #2 applying the C method assuming the point #1 as the point at the free end and point #2 as the point in the media. And it is possible to estimate the wave propagation time between the free end and the reflecting plane applying the E method with waveforms at each point. The latter can estimate the wave propagation time at seven points on the foundation as points #1 to #7 and it is expected to know about the homogeneity on the materials of the upper foundation confirming with the behavior of the estimation result at each point. Here considers only about the R direction because it is possible to exist stably only the P wave motion of the R direction as orthogonal direction against the circumference considering a situation of the wave motion field.

4. Estimated result and discussion

Table 1 shows the estimated wave propagation velocity of each direction by the R method arranging with each set of the measurement. This table also shows the estimated maximum wave propagation time and cross-correlation coefficient of that time for each direction of each set. The V direction of each set corresponds to the SV wave or the bending wave, SB wave. The T and R directions of the horizontal component correspond to the P wave and SH wave, respectively, for the sets of #0 and #1, and #8 and #7, and SH wave and P wave, respectively, for the other sets. In any case, the cross-correlation coefficient becomes small in order of P wave, SH wave and SV wave or SB wave. The waveforms of two points show high correlativity as exceeding 0.9 of the coefficient during the P wave. Here, “set of #A and #B” means the propagation from the point #A to #B in case of the plus value of the time difference, and, conversely, that from the point #B to #A in case of the minus value of the time difference.

Table 1. Estimated results applying the R method

Combination of points	#2 and #1			#2 and #3			#4 and #3			#4 and #5		
Wave type	Vsh	Vp	Vsv or Vb	Vsh	Vp	Vsv or Vb	Vsh	Vp	Vsv or Vb	Vsh	Vp	Vsv or Vb
Correlation at Max.	0.9014	0.9902	0.8170	0.7459	0.9224	0.7706	0.9284	0.9463	0.8556	0.8971	0.9824	0.7516
Max. travel time in 0.01s	-0.0643	-0.1042	-0.1927	1.1558	0.7346	1.6596	-0.0270	-0.1397	0.0206	0.3814	0.4158	1.8049
Min. travel time in 0.01s	-0.5418	-0.2406	-1.4053	-0.1575			-0.4676	-0.5940	-0.7319	-0.2133	-0.0398	
Correlation at Min.	0.9264	0.9941	0.8511	0.7135			0.9178	0.9543	0.9067	0.9315	0.9727	
Distance in m	10.7	10.7	10.7	9.4	9.4	9.4	10.6	10.6	10.6	11.2	11.2	11.2
Component	T	R	V	T	R	V	T	R	V	T	R	V
Max. correlation	0.9264	0.9941	0.8511				0.9178	0.9543	0.9067			
Estimated Velocity in m/s	-1975	-4448	-761				-2267	-1784	-1448			
Estimated Velocity in m/s				813	1280	566				2937	2694	621
Max. correlation				0.7459	0.9224	0.7706				0.8971	0.9824	0.7516

Combination of points	#6 and #5			#6 and #7			#0 and #1			#8 and #7		
Wave type	Vsh	Vp	Vsv or Vb	Vsh	Vp	Vsv or Vb	Vp	Vsh	Vsv or Vb	Vp	Vsh	Vsv or Vb
Correlation at Max.	0.9183	0.9612	0.8540	0.5437	0.7049	0.8632	0.9083	0.9311	0.9005	0.7818	0.6568	
Max. travel time in 0.01s	-0.0801	-0.0707	-0.0420	0.9051	1.5803	1.7998	0.2639	0.3123	0.8335	0.4200	0.6258	
Min. travel time in 0.01s	-0.5783	-0.6617	-0.8434	-0.0532	0.4689		-0.4665	-0.1961	-0.5695	-0.7510		-2.7986
Correlation at Min.	0.8871	0.9467	0.8745	0.7745	0.5752		0.8926	0.9440	0.9236	0.8247		0.6223
Distance in m	10.9	10.9	10.9	7.1	7.1	7.1	10.1	10.1	10.1	11.5	11.5	11.5
Component	T	R	V	T	R	V	T	R	V	T	R	V
Max. correlation	0.8871	0.9467	0.8745				0.8926			0.8247		0.6223
Estimated Velocity in m/s	-1885	-1647	-1292				-2165			-1531		-411
Estimated Velocity in m/s				784	449	394	3827	3234	1212	2738	1838	
Max. correlation				0.5437	0.7049	0.8632	0.9083	0.9311	0.9005	0.7818	0.6568	

Without some exceptions, the result of the estimation basically shows the propagation time corresponding to one direction. It is inferable that the wave motion propagates mainly from the arena, or northern side, in case of the direction of wave propagation at the points #1 to #7 on the foundation. In case of the sets of #0 and #1, and #8 and #7 as the measurement line of east-west direction, bidirectional wave propagation can be confirmed. It may be related to the location of the wave source as road traffic at three directions on north, east and south, subway at north (under construction) and west or tram at east and south as Figure 9.

Figure 9. Traffic noise sources around Colosseum

Figure 10 shows the estimated wave propagation velocity corresponding to the middle of the set of measurement point. The wave propagation velocity of the SH wave becomes higher than that of the P wave at the section including the point #4 and #5 from this figure. It suggests that the SH wave of microtremor mainly propagates from high velocity layer in underground, maybe lower foundation, to each point and there is little constituent propagating the surface layer.

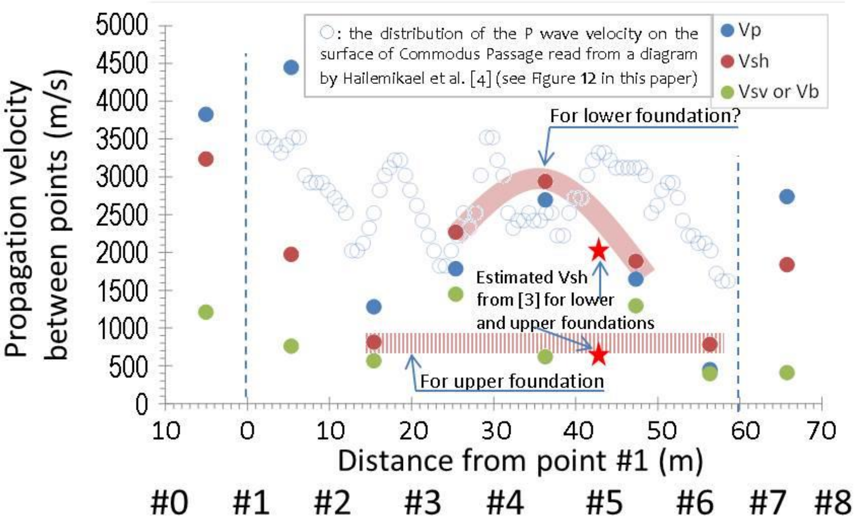


Figure 10. Estimated wave propagation velocities by the R method

On the P wave, it seems that there is little propagation from underground considering the direction of propagation and vibration and it mainly consists of direct wave propagating as the P wave in the surface layer. If there is high velocity layer behind the wall, the SV or SB wave may behave mainly as the wave propagation from the layer with relationship between the direction of the vibration and propagation. However, because there is basically upper foundation behind the wall and the propagation velocity of the upper foundation seems to be lower than that of the lower foundation, the estimated propagation velocity can be considered consequently as the SV or SB wave propagating on the floor. So, if there is cavity under the floor, it is expected that the SB wave becomes slower at such point. Additionally, in case of the wave propagation time of the SH wave for the set of #6 and #7, because the cross-correlation coefficient between the waveforms of each point becomes extremely low, there is a possibility that the propagation time is not estimated with proper accuracy. Although it is possible from the P wave velocity to estimate that the foundation close to arena is very hard, the P wave velocity at the main part of the upper foundation is almost less than 2000 m/s without rather high value between the points #4 and #5 slightly outer than the center. The SH wave distribution suggests the existence of the high velocity layer in underground around the site between the points #4 and #5 because of higher propagation velocity than the P wave around there. This portion is where the structure still remains and it can be considered that the wave propagation velocity becomes large because of the affection of confined stress.

Figure 11 shows the estimated SH wave velocity at the both foundation using microtremor Nakamura et al. [3]. The red marked point is corresponding to the measurement point #5 of Commodus passage, and at this point, the SH wave velocity was estimated 600 m/s for the upper foundation and 2000 m/s for the lower foundation. The result agrees with that of measurement in this time.

And it is confirmed that the wave propagation velocity of the SV or SB wave becomes significantly lower than that of the other wave component, between #0 and #1, #1 and #2, #4 and #5, and #7 and #8. Because it is estimated that the wave propagation velocity of the SV or SB wave becomes generally low when a cavity exists under the

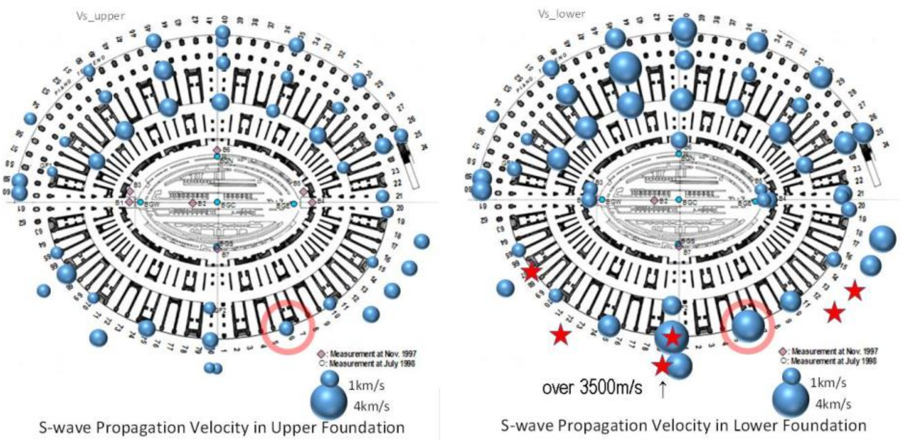


Figure 11. Estimated SH wave velocities in upper and lower foundation of Colosseum after Nakamura et al. [3]

floor, it suggests the existence of a cavity, horizontal crack or a soft layer under the floor around these points.

Figure 12 shows a longitudinal section of the P wave velocity along Commodus Passage derived from a Vp tomographic image based on the inversion of the first arrival traveltimes by Salomon Hailemikael et al. [4]. This analysis was conducted as reading totally 1395 wave arrival time of stacked waveforms in 1/1000 second accuracy with waveforms recording vibrations three times hitting the floor every 2 m using 5 kg-hammer with installing 59 vertical sensors (geophone) on

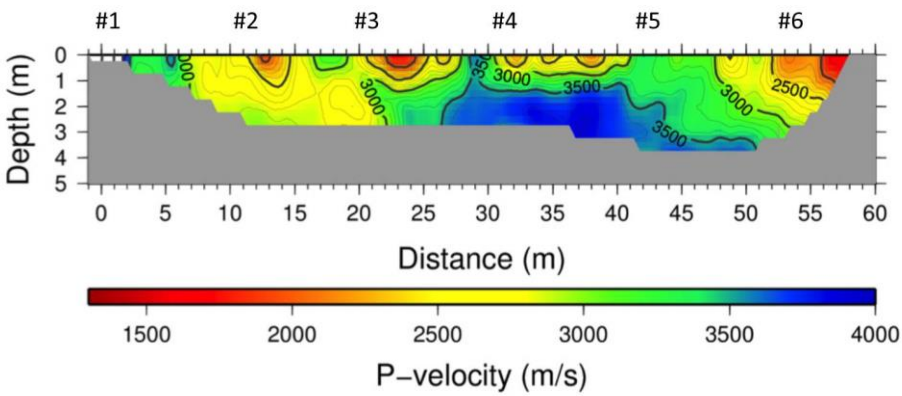
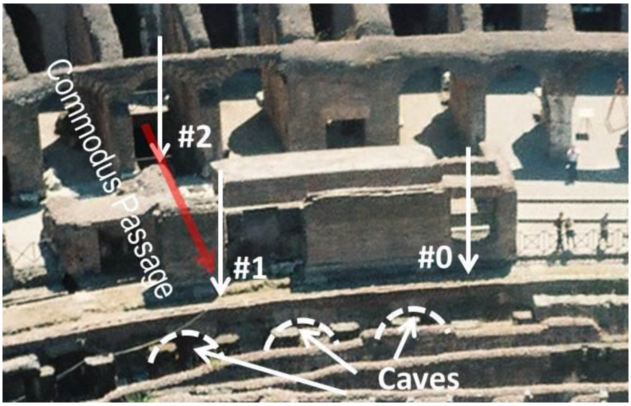


Figure 12. 2D tomographic model obtained by the seismic active data acquired along Commodus Passage [4]

the center of the floor for every 1 m. It can be confirmed from this figure that there exists high velocity layer at the underground around the point #4 or #5 and the research estimates the P wave velocity as 3500 to 4000 m/s and it is agreeable with the estimated SH wave velocity in this research as 1900 to 2900 m/s. The P wave velocity around the floor surface read from Figure 12 is shown on Figure 10 as a light colored circle and is also agreeable with the result of estimation in this research although differing in detail.

There are caves at underground between the points #0 and #1 as Photograph 2. And exploration result using the ground penetrating radar (GPR) by Luciana Orlando et al. [2] confirms an existence of layer constructions around the point #2 and in the underground between the points #4 and #5, and a similar construction in the underground between the points #7 and #8 to that between the points #1 and #2. These results agree with the result of the analysis applying the R method for microtremor.



Photograph 2. A view around the entrance of Commodus Passage

Next, the result of the wave propagation velocity estimation applying the C and E methods will be described. Figure 13 compares the propagation time or distance attenuation factor every time with relative error. Estimating the waveform as in the medium at the point #2 from the waveform observed at the point #1 as the free end applying the C method for the measurement set of #1 and #2 with distance of 10.7 m, the wave propagation time and distance attenuation can be read as 0.724 (in 1/100 sec) and 11.5/1000 (in 1/m)

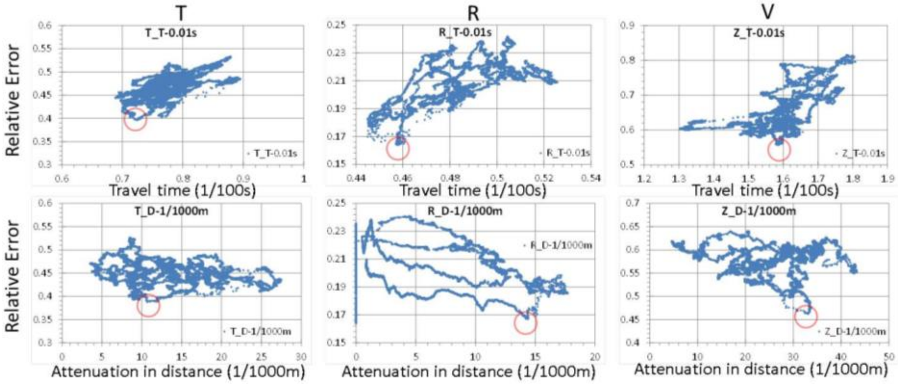


Figure 13. Relationship between travel time or attenuation and relative error derived from the simultaneous record at the points #1 and #2 applying the C method assuming the minimum relative error as the estimated value

for T direction corresponding to the SH wave, 0.458 (in 1/100 sec) and 14.4/1000 (in 1/m) for R direction corresponding to the P wave and 1.59 (in 1/100 sec) and 33/1000 (in 1/m) for V direction maybe corresponding to the bending wave, as the minimum cross-correlation between the estimated and observed waveform. It is estimated from this result that the SH wave velocity is 1480 m/s, the P wave velocity is 2340 m/s, and the bending wave velocity is 673 m/s. it is considered that the velocity estimated by the C method averagely shows the behavior of entire the layer and the velocity estimated by the R method reflects the local behavior.

And, although not indicating here, because the predominant frequency of the R direction is around 10Hz, the wavelength predominating at the wave field seems to become over 200 m. This wavelength is considerably longer than the dimension of the foundation as 12 m thick and 60 m width, and the measured P wave velocity can be considered close to that in a thin rod instead of that in an infinite media. Assuming the measured P wave velocity between the points #1 and #2 as that in a thin rod, Poisson’s ratio of these section is estimated to be 0.250 in relation to the measured S wave velocity. Additionally, assuming as in an infinite media, Poisson’s ratio becomes 0.167. In either case, it is interesting that they are in a same range of Poisson’s ratio of modern concrete.

The propagation time estimated by the E method can be considered as giving the wave propagation time between the free end (the arena side end of the foundation plate) and the reflecting plane (outer side of the foundation plate), and this research considers on the radial direction corresponding to the P wave. Figure 14 shows the propagation velocity of the P wave horizontally propagating in the foundation plate estimated at each measurement point. Here, the P wave propagation distance is assumed as the distance excluding the width of the retaining walls at the end of the arena and periphery of the foundation from the width of entire the foundation, 53 m from Figure 5. These retaining walls had worked as a formwork when constructing Colosseum. The P wave velocity between the points #1 and #2 derived from the C method is indicated as a mark "★" in Figure 14 and the velocity agrees with that derived from the E method. What the wave propagation velocity becomes higher toward out side is considered that the measured floor descends gradually after the point #3 and approach the lower foundation.

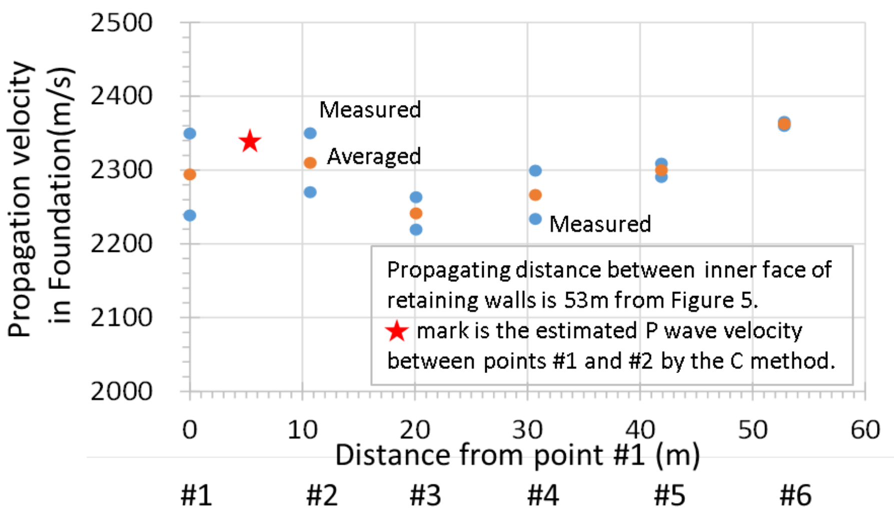


Figure 14. Estimated P wave propagation velocities on the radial direction at each point applying the E method

5. Conclusion

Microtremor was measured at the floor of Commodus Passage constructed by open-cut the foundation of Colosseum and the physical properties of the foundation were confirmed. As a result, it can be estimated the wave propagation velocity of various wave motion component for every about 10 m on the floor. This result is agreeable with the P wave velocity structure by Salomon Hailemikael et al. [4] or the search result using the ground penetrating radar (GPR) by Luciana Orlando et al. [2]. It is expected that the CERS methods make possible in the future to confirm the physical properties of entire the foundation with simultaneous microtremor measurement not only at horizontally distanced two points but also at vertically distanced two points.

Acknowledgement

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