Dynamic Characteristics of the Colosseum at the Pillar #40 Comparing the Results of Microtremor Measurement in 1998 and 2013

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Abstract

Our first extensive microtremor measurement at Colosseum was in 1998, and we had an opportunity to measure again near the pillar #40 in 2013 at almost the same point of the past measurement. Here, the results of the measurement in 1998 and 2013 at the pillar #40 can be compared. As a result, both spectral shapes basically agree well in wide frequency range. But with confirming the detail of the predominant frequency and the amplification factor, they slightly differ for each other. The result of measurement in 2013 shows that the peaks shift to a little higher for the radial direction and lower for the tangential direction, and the amplification factor becomes larger for each direction. And the mode changing the phase for 180 degrees between 3F and 4F can be commonly observed in 1998 but not in 2013. There seems to be something structural difference between 3F and 4F, for example the affection of the 2009 L'Aquila earthquake or the work of the floors and the fences of 4F and 3F in 2010.

If the degradation of the structure can be grasped quantitatively with periodic microtremor measurement, it is possible to maintain rationally and is not impossible to take a countermeasure as reinforcement of maintenance prophylactically and properly. It is necessary for establishing the application method to confirm the accuracy of the microtremor measurement, and to make clear the reason of changing the dynamic characteristics at pillar #40 quantitatively.

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1. Introduction

The Colosseum is a huge amphitheater which Roman Emperor Vespasian built to amuse common people on a reclaimed artificial lake at the garden of Nero's Domus Aurea as if to wipe away the memory of nightmare caused by Nero a tyrant from common people [1, 2, 3]. Plane plan is ellipse with 188 m of long axis, 156 m of short axis and 49 m of height. Although there are different stories, it had been started construction during the early part of AC 70's and completed at AC 80 of next Emperor Titus era. Because huge structure was built on a soft ground at a valley enclosed by hills on three sides, extremely hard foundation had been constructed. A discotic foundation with more than 12 m of thickness had been constructed by two layers made with ancient concrete, or Roman concrete, and the shape is like a donut because of avoiding the area as a field.

Huge construction of Colosseum was built on this hard foundation, but as a result of keeping exposition of the natural or artificial action for a quite long time near 2000 years, it is suffered seriously damage although the extensive repair works was done repeatedly. However the foundation has remained, a half of outer upper structure had been lost by the 1354 and other earthquakes. Additionally some part had been moved away as a stone material for other structures at the Middle age.

At the survived area, the outer wall has still remained almost to the top, but the inner structure has remained almost to the second floor, 2F, and partially fourth floor, 4F, the top floor. Upper structure is supported by pillars from ground floor, GF, to 2F in appearance, and there is wall over third floor, 3F. The each side of the survived structure was reinforced in early 19th century, and the eastern and western part is called Stern and Valadier from the name of conducting engineers to-day, respectively.

In these recent days, the environment surrounding Colosseum has been changed largely and the affection of the vibration caused by social activity, so called traffic vibration has been concerned. Additionally the subway Linea B constructed after the World War II, there are road and tram line passing the northern to the western side, and the new subway Linea C is recently under construction in the northern side [4]. Under this situation, a restoration project has been started by a large sum of donation by a benefactor since 2013 [5]. The restoration is seems to be not for changing the actual condition but for consolidation with reinforcing or protecting the degraded or weakened parts properly without changing the actual situation with the purpose for mainly cleaning.

Aside from the impact investigation of the vibration caused by the existing subway, a research group considers what kind of vibration will be caused by the new subway line construction and affect Colosseum, and how to decrease the influence. And they discuss about the vehicle as not only the iron wheel type but also rubber tire type or magnetic levitation type. A large scale FEM numerical model was constructed for this consideration [6]. Because the model requires accurate physical parameters, results of past investigations on Colosseum are gathered and additional investigations are planned.

The first microtremor measurement on Colosseum was conducted by Clemente et al. in 1985, and also other measurements has been reported [7, 8]. Among those measurements, our measurement in 1998 is recognized as the systematic distribution of the largest number of measurement point. We have reanalyzed the data and kept conducting additional measurement for the ground around Colosseum in a chance of visiting Rome with the view of this situation.

And we could measure again partially not only at the surrounding ground but also at the underground area of the arena and the main body of Colosseum in 2013. These measurements give almost same result and there is no other result of measurement at precisely the same point with time distance. But in 2013 we could measure, likely as a measurement in 1998, at almost same points with almost same instruments in the same procedure as simultaneously measurement at two points, so we can consider the difference of microtremor characteristics measured in a time span of 15 years.

This must be the first attempt to substantiate that this kind of measurements can make clear the degree of degradation quantitatively by conducting the same kind of measurements in the future with clarifying the measuring error, the statistical scatter or other parameters.

Our first measurement of Colosseum was on November, 1997. After this preliminary measurement, we conducted the first extensive measurement on July, 1998. The results of these measurements have been opened in some conferences [9, 10]. Here describes the result of comparison of microtremor characteristics at each layer measured on July 7, 1998 and on December 13, 2013 at the pillar #40 of the outer ring, with the numbering system of totally 80 pillars of the outer ring as counterclockwise rotation from the south pillar as Figure 4 of [11].

2. Measured Points and the Procedure of Measurement and Analysis

Figure 1 shows the measured points in 1998 and 2013. Figure 2 shows the location of the pillar #40 and the location of microtremor measurements in 2013 on the floor plan picture. And Photo 1 shows the situation of the measurement points in 1998 and 2013. Although both measurement were conducted almost at the same points, the location of the point at GF differs for each other. The location was outside



Figure 1 Location of measurement points at pillar #40 of the Colosseum



Figure 2 Measurement points in 2013 (small greens are measured in 1998)

the pillar in 1998 and inside the pillar in 2013 because of the difference of the public area configuration.

In 2013, microtremor of three directions, tangential direction T, radial direction R and vertical direction V, was recorded in 1/100 seconds of sampling time with a fixed instrument at the highest floor, 4F, and with a moving instrument at 4F, 3F, 2F, 1F or GF. This means that this measurement recording the data simultaneously at two points including the fixed point is devised to get a same result of simultaneous measurement at the entire point with same instruments. The measurement in 1998 was also conducted simultaneously at two points with three components, R, T and V, but the characteristics of the instrument were not complemented.

The measurement in 1998 recorded microtremor as a 40.96 seconds data, 4096 samples, repeating three times, and that in 2013 recorded microtremor as a five minutes data at each point, chose five sections of 40.96 seconds data, 4096 samples, and then averaged the five data sections with frequency analysis. Earthquake motion response characteristics of each floor based on GF are derived for each measurement.



Photo 1 Measurement points at pillar #40 in 2013 and 1998 Red circles: sensor location, Blue triangle: sensor location in 2013

And K_b value is derived from the earthquake motion response characteristics. This K_b value is an index to indicate the vulnerability of the structures focusing the drift angle and is formulated as follows [10] (see Figure 3).



Assuming that the predominant frequency dominates and each floor vibrates as the vibration mode of the predominant frequency, then the response displacement δ_i of *i*th floor against the earthquake acceleration α from GF becomes as flows;

$$\delta_{\rm i} = A_{\rm i} \alpha / (2\pi F)^2 \tag{1}$$

Figure 3 Mode shape of structure

Here, A_i is the amplification factor of *i*th floor. Then, the drift angle between *i*th and *j*th floors γ_i at *j*th floor becomes as follows;

$$\gamma_{j} = |A_{i} - A_{j}|\alpha/(2\pi F)^{2}/H_{j} = |A_{i} - A_{j}|/(4\pi^{2}F^{2}H_{j}) \times \alpha = K_{bj} \times \alpha$$
(2)

Here, H_j is the height of the pillar at *j*th floor, |*| is an absolute value of *, and j = i + 1.

The K_{bj} value can be considered as an index relating to the vulnerability of this pillar. With setting the unit of the drift angle to 10⁻⁶, a height of a pillar in m and acceleration in Gal (= cm/sec²) the K_{bj} value can be expressed as follows;

$$K_{bj} = 10^4 \times |A_i - A_j| / (4\pi^2 F^2 H_j)$$
(3)

This is the K_{bj} value for the vulnerability of a structure. The predominant frequency and the amplification factor for each floor can be derived from the result of the microtremor measurement. Although the estimation based on the microtremor is only a result of the response at a minute level and does not suppose a large deformation, it is considered to be able to order the measured points by the vulnerability.

As mentioned above, the K_{bj} value estimated from the result of the microtremor and the dimension of the structure is expected to order the structure with the vulnerability of each structure, the component or the floor, and to clarify the weak points. This paper calculates the K_{bj} value for comparison of the measurement result at each time and location.

3. Results of Analysis

Figure 4 shows the result of the comparison between the measurement in 1998 and in 2013. The shape of the transfer function well agrees for each measurement at a wide frequency range for the three components. However noticing to the detail of the predominant frequency and the amplification factor, they differ slightly for each other. Table 1 shows the predominant frequency and its amplification factor. It can be seen that the predominant frequency of the measurement in 2013 shifts higher for R component and lower for T component than that of the measurement in 1998. The amplification factor of the measurement in 2013 becomes larger than that in 1998 for each direction. That of V component changes slightly large and the predominant frequency seems to shift higher.

2013		Height(m)	Tangential	Radial	Vertical	Kbi (µ	<mark>/Gal)</mark> Tangantial		Padial	Vortical
Predom. Fre. (Hz)			2.661	1.733	13.330	between	Height(m)	rangential	radiai	vertical
Amplification against GF	4F	37.5	16.7	23.9	7.3	4F and 3F	34.79	2.4	98.5	0.20
	3F	32.08	16.3	17.5	6.5	3F and 2F	28.02	19.5	75.7	0.30
	2F	23.96	11.9	10.3	4.8	2F and 1F	18.215	19.0	51.4	0.28
	1F	12.47	5.8	3.2	2.5	1F and GF	6.235	13.8	15.2	0.18
	GF	0.0	1.0	1.0	1.0					
						Kbi (μ/Gal)		Tanankal	Deskal	Marthaut
1998		Height(m)	Tangential	Radial	Vertical	Kbi (µ	l∕Gal)	T	Destat	Vesteral
1998 Predom. Fre. (Hz)		Height(m)	Tangential 2.905	Radial	Vertical 10.547	Kbi (µ between	<mark>∕Gal)</mark> Height(m)	Tangential	Radial	Vertical
1998 Predom. Fre. (Hz)	4F	Height(m) 37.5	Tangential 2.905 14.1	Radial 1.685 18.5	Vertical 10.547 5.3	Kbi (µ between 4F and 3F	<mark>/Gal)</mark> Height(m) 34.79	Tangential 8.1	Radial 71.3	Vertical 0.16
1998 Predom. Fre. (Hz)	4F 3F	Height(m) 37.5 32.08	Tangential 2.905 14.1 12.7	Radial 1.685 18.5 14.1	Vertical 10.547 5.3 5.0	Kbi (µ between 4F and 3F 3F and 2F	<mark>/Gal)</mark> Height(m) 34.79 28.02	Tangential 8.1 12.5	Radial 71.3 61.9	Vertical 0.16 0.26
1998 Predom. Fre. (Hz) Amplification	4F 3F 2F	Height(m) 37.5 32.08 23.96	Tangential 2.905 14.1 12.7 9.3	Radial 1.685 18.5 14.1 8.5	Vertical 10.547 5.3 5.0 4.0	Kbi (µ between 4F and 3F 3F and 2F 2F and 1F	/Gal) Height(m) 34.79 28.02 18.215	Tangential 8.1 12.5 12.2	Radial 71.3 61.9 42.8	Vertical 0.16 0.26 0.35
1998 Predom. Fre. (Hz) Amplification against GF	4F 3F 2F 1F	Height(m) 37.5 32.08 23.96 12.47	Tangential 2.905 14.1 12.7 9.3 4.6	Radial 1.685 18.5 14.1 8.5 3.0	Vertical 10.547 5.3 5.0 4.0 2.3	Kbi (µ between 4F and 3F 3F and 2F 2F and 1F 1F and GF	/Gal) Height(m) 34.79 28.02 18.215 6.235	Tangential 8.1 12.5 12.2 8.7	Radial 71.3 61.9 42.8 14.4	Vertical 0.16 0.26 0.35 0.24

Table 1 Predominant frequency and its amplification factor





According to the structure of Colosseum, because especially the T component and the V component of the higher layer are the direction along the wall, it is estimated that the degradation of the structure is easy to be seen. Meanwhile, because the R component is perpendicular to the wall and affected by the bending, this component is seems to be sensitive for the local effect. On the other hand, because the sensor is set at a point on a floor for measurement, leaving from the wall makes sensitive against the vibration of the floor. Especially in case of the vertical motion, it is impossible to ignore the effect when the rigidity of the floor is poor. However looking at the cross-section as shown in Figure 1, the rigidity of the floor close to a wall or a pillar is rather high because the floor is supported by an arch structure. So the vibration of the floor seems to have only a small impact. Although the GF point was set outer and inner side of the pillar #40 for each measurement as described previously, both the dynamic characteristics with a distance of 2 m are basically not much difference because the points are on the hard ancient concrete foundation 12 m thick. We would like to verify precisely the effect caused by the difference of the measurement point for the result of the analysis in the future. See Photo 1 to grasp the difference of the location in detail.

From the result of the measurement in 2013, the predominant frequency of R component becomes higher than that in 1998 and that of T component becomes obviously lower than that in 1998. The obvious decreasing the predominant frequency of T component is feared to be a phenomenon relating to a structural problem, because the horizontal components seem to be little affected by the location of the measurement point.

Figure 5 shows a vibration mode of the frequency maximizing the amplification factor at 4F. There can be seen a lot of other peaks on the transfer function. Especially for the vertical motion, the peak is at high frequency range and corresponding roughly to each measurement in 1998 and 2013, however it has a possibility to be not corresponding precisely.

The vibration mode at various frequencies is animated with sweeping the frequency to understand precisely the vibration mode at the peak frequency. Please see this animation on our website [12]. From the animated mode diagram shown in Figure 6, there are many modes changing the phase 180 degrees between 3F and 4F at R component increasing the predominant frequency for the measurement in 1998, however these modes are little seen for that in 2013 measurement. It suggests that there must be some structural changes be-



Figure 5 First modes for each component at pillar #40



Figure 6 An Example of animated mode diagram from the animation

tween 3F and 4F during 1998 to 2013. During this period the 2009 L'Aquila Earthquake M6.3 occurred at around 90 km in the northeast of Colosseum. It is estimated that Rome might experience the earthquake motion of Realtime Intensity RI 3, almost same as the JMA intensity [13], corresponding to MMI 5, but there is no report on the serious damage for Colosseum.

Table 1 also shows the K_b value of the structure derived from the predominant frequencies and their amplification factors. Figure 7 shows the vertical distribution of derived K_b value. Comparing the K_b value at each component, R component shows predominantly large value and it suggests this component is weakened. The value for T component is around a fifth of that for R component. In response of the value for R component becoming lager for upper layer, that for T component is small at GF and 3F and large at 1F and 2F. The value for V component reaches maximum at 1F or 2F but the value is considerably small. The distribution of the K_b value for T and V components is thought to reflect the situation becoming less deformable by the wall structure over 3F. Almost all the component shows lager value for the measurement in 2013 against that in 1998. Despite the decreasing the predominant frequency, it becomes the largest for R component and the value corresponding to reaching 1 % of the story drift against the earthquake motion of 100 Gal at the wall of 3F. The



Figure 7 K_b value distribution for the first modes at pillar #40

value for T component of the measurement in 2013 becomes 1.4 times larger than that in 1998. It is startling that such a large change is occurred at most 15 years. It is considered that the result of the analysis may include not only the measurement error but also the statistical error for the measured data.

As mentioned above, the change of the mode, the predominant frequency, the amplification factor and the K_b value was unpredictably large between the measurement in 1998 and 2013. So we tried confirming a change on appearance around the measurement points based on the movies and pictures during the measurement. Photo 2 shows the situation around the point 4F both in 1998 and 2013.

At first focusing the floor setting the sensor, it is confirmed that there is an original travertine pavement at GF, a concrete pavement between 1F and 3F, and a mortar pavement with brick pieces at 4F. The pavement between 1F and 3F seems to be mainly casted before 1998 with unknown thick. The floor contacts the wall at 3F and 4F. A concrete is casted with a distance 15 cm around the pillar at 1F and 2F. Although there are mainly radial joints, there were concentric joints at 4F in 1998 as Photo 2. But in 2013, the concentric joints changed to radial joints. And the fences were also changed clearly between 1998 and 2013 at 4F. It is possible to confirm the concentric joints on the picture of Google Earth dated July 29, 2007, but the situation of the floor was clearly changed on November 10, 2011 from the background picture of Figure 2. And the shadow of the fence on the floor at that time suggests that it was same as the one at the time of the measurement in 2013. So it is possible to say that a work refurbishing the floor and the fences of 4F between July 29, 2007 and November 10, 2011. Finally the subsequent investigation made clear that work for 3F and 4F conducted between March and November 2010 shown in Photo 3.

The pavement of the 4F floor in 1998 was separated mainly in three sections by two concentric joints to the fences. There are concentric joints on the pavement contacting the wall and it seems to be unified totally to the wall.



Photo 2 Comparison of measurement situation around pillar #40 between in 1998 and in 2013



Photo 3 Old and new fences appeared in photos

On the other hand, the 4F floor in 2013 was separated with a distance 2 m to 3 m by radial joints from the wall to the fence. Although the strength of the floor pavement is unclear, assuming the floor pavement connected tightly by the joints, the 4F floor pavement in 1998 and 2013 seems to have an effect to reinforce the tangential T component and the radial R component from the direction of the joints, respectively. It is agreeable that the predominant frequency of T direction becomes lower in 2013, that of R component becomes higher in 2013, and the phenomenon reversing the vibration mode of R component at 3F and 4F is confirmed in 1998 and not in 2013. The measurement points without GF locate at an area not opened to public and then there seems to be no degradation for the floor pavement caused by passing tourers, but the floor concrete at 1F and 2F shows degradation clearly as expanding opening with increasing cracks on the floor pavement or exfoliation and fracture of joints. However it is difficult to confirm the situation of the degradation clearly because of the dark image at 3F, a structural factor of the degradation may exist. Meanwhile, the fence itself and its location at 1F and 2F differ for the measurement in 1998 and 2013 as shown in Photo 1.

The changes above are impressive because of the relation to the change of the dynamic characteristics derived from microtremor. We would like to grasp quantitatively the relation to the change of the microtremor characteristics with confirming the description of work at 4F and grasping the detailed structural characteristics. We expect that the microtremor measurement will be applied as a powerful tool for reasonable maintenance of the structure.

4. Conclusion

This paper considered the change of the dynamic characteristics of Colosseum with comparing the result of the microtremor measurement at the pillar #40 in 1998 and 2013. As a result, an unexpected large change was grasped for the predominant frequency and its amplification. The factors for this change are considered as the effect of the 2009 L'Aquila Earthquake, the effect of the work modifying the 4F floor and the fence, and additionally the factors on the measurement itself as the measuring or the statistical error. This comparison made

confirm specifically the possibility that the measured microtremor characteristics reflect the change of the structural characteristics, so it seems to be necessary to consider precisely making clear the impact rate of each factor as a future issue.

It is important for reinforcing the structure of Colosseum to grasp the stiffening effect quantitatively. So it is necessary to investigate before and after the reinforcement. Additionally it is possible to maintain the structure reasonably and not impossible to provide preventive and proper countermeasures for reinforcement or conservation, if the degradation of the structure can be grasped quantitatively with a periodical microtremor measurement. It is necessary for establishing this utilization method to confirm the accuracy of the microtremor investigation and also to explain quantitatively the reason changing the dynamic characteristics of the pillar #40.

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