Dynamic Characteristics of Memorial Columns of Trajan and Marcus Aurelius using Microtremor

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Abstract

Memorial Columns of Trajan (TC) and Marcus Aurelius (AC) in Rome with a similar structure and size were studied their dynamic characteristics by microtremor measurement. As a result, the fundamental frequencies of horizontal vibration are approximately 1.6 Hz for TC and 1.3 Hz for AC, and caused mainly by bending vibration. These agree with the result of the previous investigation. Predominant frequencies of torsion and longitudinal vibrations are estimated approximately 15.5 Hz and 25.6 Hz, respectively. From these values the propagation velocities of shear wave and longitudinal vibration are estimated as 1900 m/s and 3100 m/s, respectively. Although these values are relatively small, about 65% of typical value, Poisson's ratio is estimated reasonable value about 0.33. On the other hand, the predominant frequencies of the torsional and longitudinal vibrations of AC are approximately 9.1 Hz and 18.8 Hz, respectively, and the velocity of shear wave and longitudinal vibration are estimated considerably low value as 1100 m/s and 2300 m/s, respectively. Longitudinal vibration propagation velocity is more than 50% lower than typical value, and the shear wave velocity is too low to estimate Poisson's ratio. It seems to be caused by low natural frequency of torsional vibration, and it is observed a tightened up vertical long damage on the west side of column body causing low registance against torsional vibration. The destructive index Kb-value of the columns derived from the fundamental mode of bending vibration is 83 μ/Gal in maximum for upper part of TC and is 212 μ/Gal for top of AC with large variation. Thus, drift angle over 1/120 radian will be caused about 100 Gal of earthquake motion at base ground for TC and lower than 50 Gal of earthquake motion for AC.

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1. Introduction

Memorial Columns of Trajan and Marcus Aurelius (referred as TC and AC hereafter) in Rome had been completed with same structure and size in 113 AD and 193 AD, respectively. Cylindrical surface of each column has spiral relief. Each memorial columns consist of stacked marble blocks hollowing out spiral staircase and each block is approxicimately 3.6m diameter and 1.52m height in average and has hollowed out 8 steps spiral staircase. Totally 19 blocks including terrace and base of the column are layered for approximately 30m height with two blocks for stature base on terrace. ***Figure 1*** shows models of staircase and block. Their foundation grounds are hard for TC and relatively soft for AC. AC has been received severe damage of weathering and earthquakes, and massive restoration has been done already in the 16th century. Although TC was made earlier than AC and

Figure 1. *Models of Spiral Staircase and Block*

has been received the weathering and artificial damages, it still remains almost original form in present.

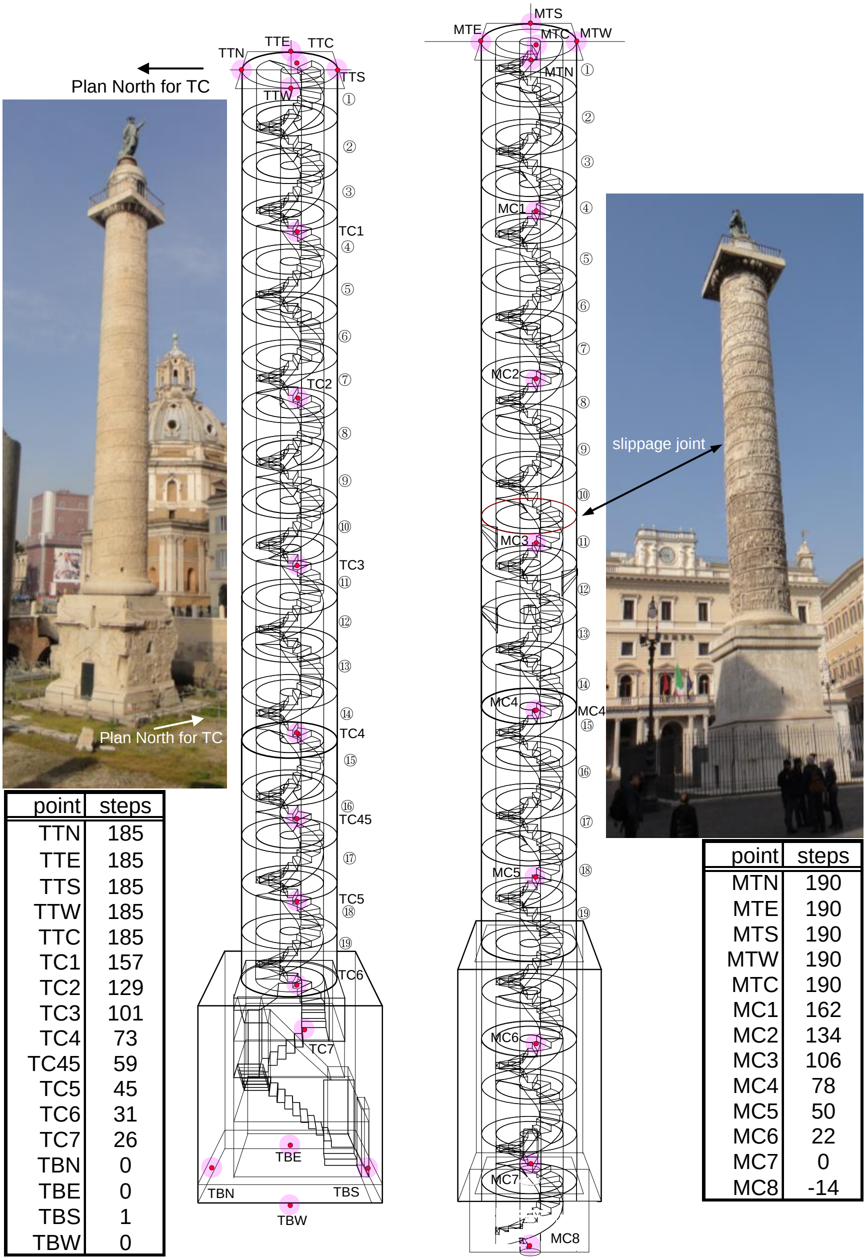
There have been research works on dynamic characteristics of TC and AC previously, and our research aims to grasp precisely the existing condition and reports the result of the preliminary microtremor measurement for both TC and AC.

2. Microtremor Measurement and Data Analysis

The measurements were done between 10 a.m. to 2 p.m. at 28th March of 2012, TC in the morning and AC in the afternoon. The weather was fine. ***Figure 2*** shows the distribution of the measuring points for TC and AC, indicating for under the terrace and spiral staircase without accuracy in dimension.

For TC there are five measurement points, TTN, TTE, TTS, TTW and TTC on the terrace, the highest floor. Two couples of measurement points, TTN-TTS and TTE-TTW, were measured simultaneously for detecting rocking and torsional vibration. Also four points on surface ground, TBN, TBE, TBS and TBW were set with contacting the pedestal, and two couples of measurement points, TBN-TBS and TBE-TBW, were measured simultaneously. For measurement of column body, one continuous measuring point TTC, inside of the entrance on east side of the terrace, was set for simultaneous measurement. The spiral staircase of TC turns one rotation by 14 steps. From TTC, four points, TC1 to TC4, were set every 28 steps, and other three points, TC45, TC5 and TC6 were set every 14 steps in the column body, and TC7 five steps under TC6 were set in the pedestal.

For AC, all the measuring points were measured individually. Five points, MTN, MTE, MTS, MTW and MTC on the terrace, under the MTC, six points of MT1 to MT6 were set every 28 steps, and MT7 and MT8 were set 22 steps and 36 steps under the MT6 respectively. The spiral staircase of AC has 14 steps per turn as same as TC. At each point, microtremor of three components EW, NS and UD were measured simultaneously for duration of three minutes with 1/100 seconds sampling.



point	steps
TTN	185
TTE	185
TTS	185
TTW	185
TTC	185
TC1	157
TC2	129
TC3	101
TC4	73
TC45	59
TC5	45
TC6	31
TC7	26
TBN	0
TBE	0
TBS	1
TBW	0

point	steps
MTN	190
MTE	190
MTS	190
MTW	190
MTC	190
MC1	162
MC2	134
MC3	106
MC4	78
MC5	50
MC6	22
MC7	0
MC8	-14

(1) Trajan's Column (2) Marcus Aurelius's Column

***Figure 2.** Distribution of the Points for Microtremor Measurements*

Frequency analysis has been carried out by Fourier transform for the measured waveform. The response spectra of TC and AC have been estimated using simultaneous and individual measurement, respectively. The analysis used 40.96 seconds (4096 data) length record and the final result was derived spectra averaged three or four times. Also the spectral ratio of target point against base point or H/V spectral ratio is derived. These results of individual or simultaneous measurement are compared and investigated.

3. Result of Analysis

3.1. Column of Trajan

Table 1 shows the results of the analysis of the predominant fre-

Table 1. *Results of Microtremor Measurement of TC*

TC	Simultaneous.	F Hz	maxA	maxKb
EW	1st	1.59	18.8	83.1
	2nd	8.99	7.8	5.0
	3rd	19.17	6.3	1.2
NS	1st	1.59	15.5	77.2
	2nd	8.99	5.2	3.4
	3rd	20.29	9.3	1.0
Longitudinal	1st	25.64	7.8	0.2
Torsional	1st	15.53		
TC	Individual	F Hz	maxA	maxKb
EW	1st	1.59	188.8	2271
	2nd	9.03	8.3	4.8
	3rd	19.04	6.0	1.2
NS	1st	1.59	98.6	1293
	2nd	9.03	7.3	4.4
	3rd	20.26	9.6	1.3
TC	H/V	F Hz	maxA	maxKb/Kg
EW	1st	1.51	22.0	181
NS	1st	1.44	11.2	111
Ground: BN	1st	5.81	1.9	0.6
BE	1st	5.00	2.0	0.8
BS	1st	5.27	2.3	1.0
BW	1st	5.00	2.3	1.0

quencies for each component and position. According to this table, comparing simultaneous and individual measurements, they are almost equal for each other on natural frequencies, but there is large difference for 1st mode shape. On the other hand, from H/V spectral ratio, estimated natural frequencies are little lower but amplification factor is close to the results estimated by simultaneous measurement. The following explains the results of these analyzes in detail.

3.1.1. Simultaneous Measurements

Figure 3 shows the amplification spectra of the sites along the column elevation against the site TC7, 26 steps from the lowest level in a pedestal as a rigid block, as a result of the simultaneous measurement. It is possible to draw the vibration mode for the entire frequency mode because the phase can be grasped by the simultaneous measurement. Please find an animation to visualize the vibration mode sweeping

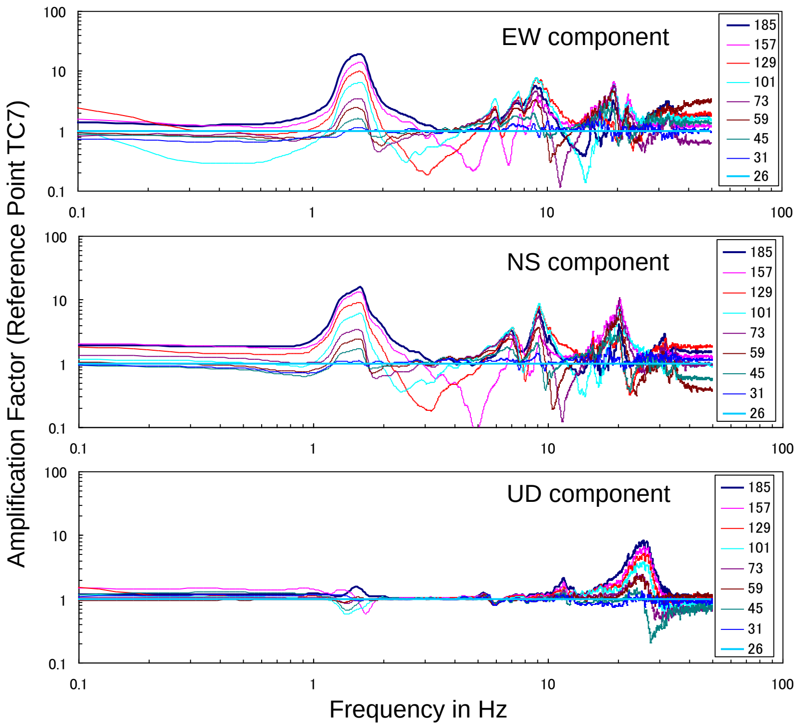


Figure 3. *Amplification Spectra of TC estimated from Simultaneous Measurements*

frequency in website ([6]).

Figure 4 shows the vibration modes for each peak frequency with considering phase difference. These figures show that the number of peaks of EW component is larger than that of NS component, that the first peak frequency is 1.59Hz for both EW and NS components, and that the shape of the mode diagram is slightly differ from each other.

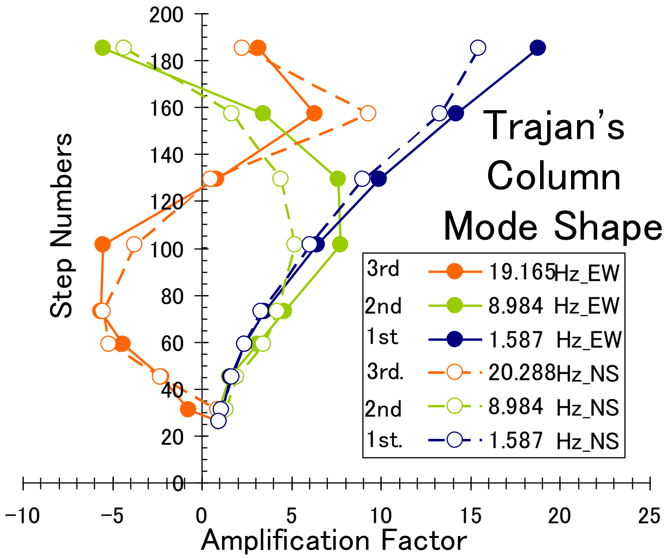


Figure 4. *Mode Shape of TC estimated from Simultaneous Measurement*

The shape of mode diagram for the first peak frequency is smooth. That of both EW and NS components seems to consist of mostly bending deformation and only that of NS component is recognized overlapped slightly shear deformation.

Second order frequencies with large peak are estimated 8.99Hz for each direction with the second order mode shape. Natural frequencies of third order are estimated 19.17 Hz for EW and 20.29Hz for NS with third order mode shape. There are many spectral peaks in response spectra. These peaks might be corresponding to the local damage of column.

For the bending vibration of a uniform cantilever, the natural frequency's ratio between first order mode and second or higher mode is approximately 1: 6.2669···: 17.5449···: ···. In case of shear or torsional vibration, this ratio becomes 1: 3: 5: ···. According to the result of microtremor measurement, it could be clearly recognized up to the natural frequency of third order mode, and the ratio is approximately 1:5.65:12.06 for EW component and approximately 1:5.65:12.76 for NS component. Consequently the horizontal vibration of TC is estimated consisting of almost bending vibration.

According to the measurements on the terrace, there is torsional vibration at the frequency range between 14Hz and 19Hz, and reaching maximum at 15.5Hz. At other frequency range torsional vibration can not be recognized. At the range between 14Hz and 19Hz, the peak and the trough of the vibration modes are interchanged intricately at each measuring points on the column body, and this phenomena may appear the effect of torsional vibration (see an animation on website of SDR, reference [6]). Shear wave velocity of TC is estimated 1900 m/s from the predominant frequency of torsional vibration.

In addition, the vertical vibration of TC predominates at 25.6Hz, and the vibration mode shape shows the first order. And the propaga-

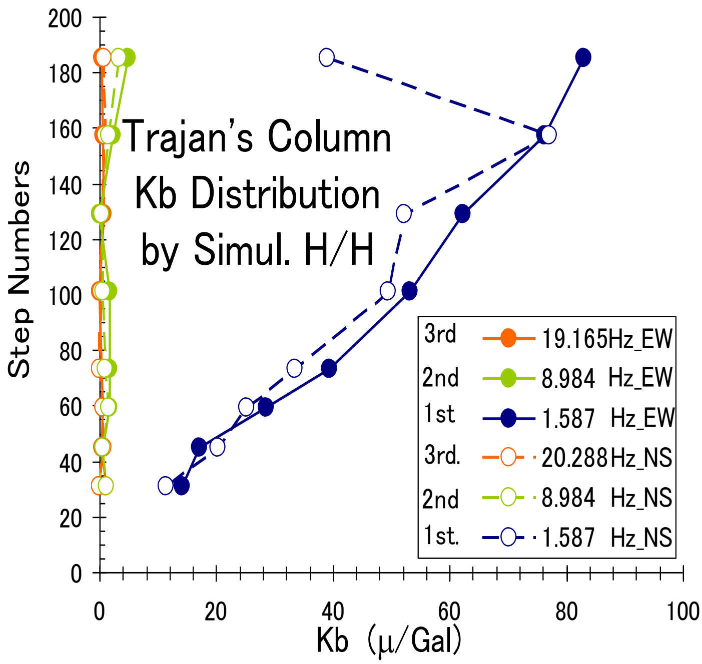


Figure 5. *Kb-value of TC estimated from Simultaneous Measurement*

tion velocity of the longitudinal vibration of TC is estimated 3100 m/s, and the Poisson's ratio is estimated 0.33 from the shear wave velocity and it is a reasonable estimation as marble. However, because the estimated velocities of TC are about 65% of the typical value of marble, 4800 m/s for longitudinal vibration or 2900 m/s for shear wave, TC seems to be considerably deteriorated.

Figure 5 shows a destructive index Kb value derived from the estimated amplification mode of horizontal vibration. Please see the detail of Kb value on reference [4] as a name of K_T. According to this figure, the influence of the first order vibration mode is very large, and it is possible to ignore the higher modes. Distribution of Kb values by the first order vibration grows higher toward to the top, and reaches the maximum value of 83μ/Gal at the top. Strong motion of PGA 100 Gal will cause 1/120 of drift angle at the top of column near terrace. The ability of shear deformation of TC which is made simply by stacking large marble blocks must be considered.

3.1.2. Individual Measurements

(1) Individual Spectral Ratio

Here the response spectrum ratio against control point is considered in case of using not simultaneous measurement record.

Figure 6 shows estimated mode shape for several orders using spectral ratio to point TC7. According to this, predominant frequencies by third mode are almost equal to the results of simultaneous measurement. On the mode shape, however, the amplification factor becomes larger close to the terrace, although higher mode shape are almost same. Temporal or special stationarity of the column may be disturbed by the activities of other investigation teams on the terrace or inside the column body.

It is expected that the response characteristics can be estimated properly from the spectrum ratio against control point under keeping stationarity in case of not simultaneous measurement. However it is obvious that disturbance of stationarity causes disturbance for the estimation result. Because the column seems to be impacted constitutionally by the shaking to upper part, it is also necessary for response spectrum estimation to reconfirm to take countermeasure as limitation

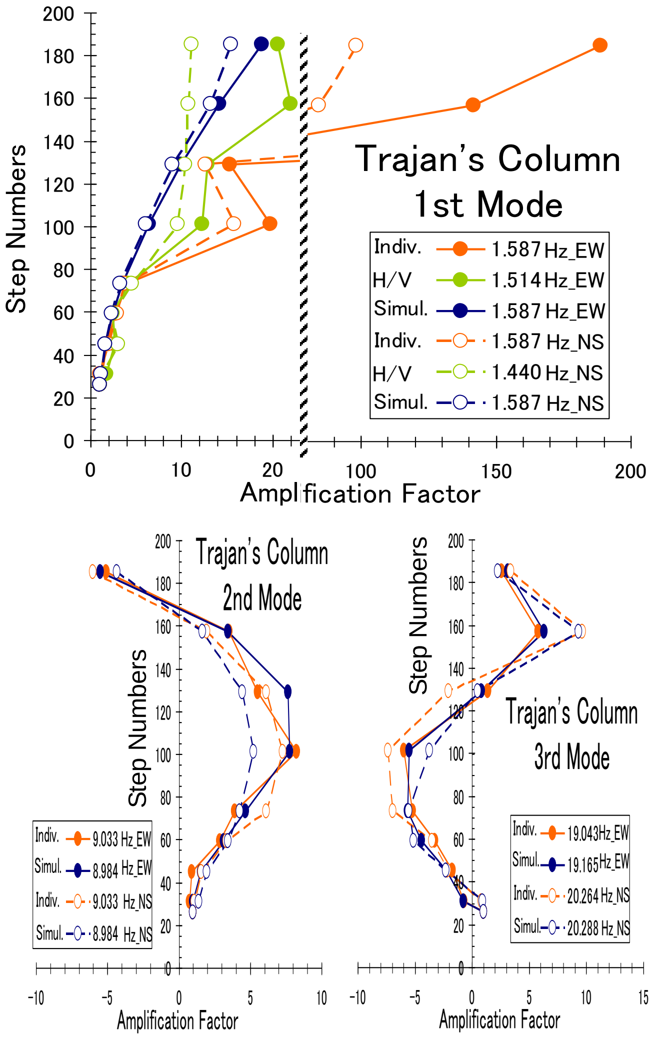


Figure 6. *Comparison of the Shape of Modes of TC between Various Methods*

of other activities during measurement or conducting simultaneous measurement.

(2) H/V and its ratio

Here, as a trial to estimate reasonable response spectrum from individual measurement using H/V spectral ratio. Response of vertical component is rather clear than that of horizontal component at high frequency range. However at lower frequency range, the characteristics of vertical motion on the base ground are similar to that of horizontal motion. This idea uses these characteristics. In case of ground, a peak appears at a frequency corresponding to the predominant frequency of the surface ground, and the peak value gives approximately the amplification factor. Also in case of structures, it is necessary to notice that the H/V spectrum ratio sometimes reflects the motion as rocking vibration.

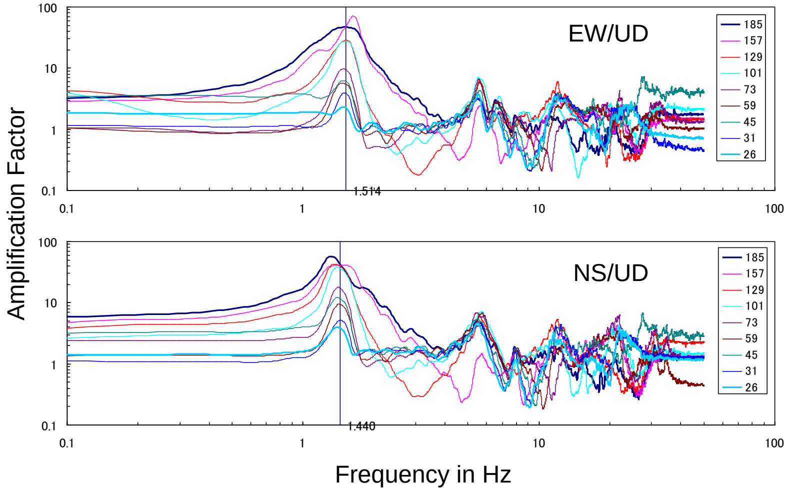


Figure 7. *H/V Spectral Ratio at the Sites of TC*

Figure 7 shows overlapped H/V of each point at TC. According to this figure, the predominant frequency of 1.5 Hz corresponds to the first order natural frequency of TC. The frequency of 5.59 Hz is recognized at all points, and it can be considered as the natural frequency for surface ground with about twice as amplification factor.

Figure 6 also shows a mode shape corresponding to the principal frequency around 1.5 Hz. According to this figure, the abnormal phenomena of enlargement of amplification factor near the terrace can be avoid, and estimated amplification factors and mode shape are similar to the results of simultaneous measurement mentioned above. In case of individual measurement, it is expected that the amplification factor and mode shape can be estimated roughly by calculating the H/V or the ratio of H/V.

3.2. Column of Marcus Aurelius

On AC each microtremor measurement was conducted individually. ***Table 2*** shows an analytical result of measurement.

***Table 2**. Results of Microtremor Measurements of AC*

AC	Individual	F Hz	maxA	maxKb
EW	1st	1.29	64.0	936
	2nd	6.59	15.2	17.9
	3rd	15.02	12.4	2.3
NS	1st	1.22	53.3	957
	2nd	6.98	19.4	15.9
	3rd	15.14	20.9	3.0
Longitudinal	1st	18.80	57.7	1.0
Torsional	1st	9.13		
	2nd	25.95		
AC	H/V	F Hz	maxAF	maxKb/Kg
EW	1st	1.25	18.8	212
NS	1st	1.22	13.9	144
Ground: MC7	1st	1.22	2.54	5.3
CMGW	1st	1.37	2.76	5.6

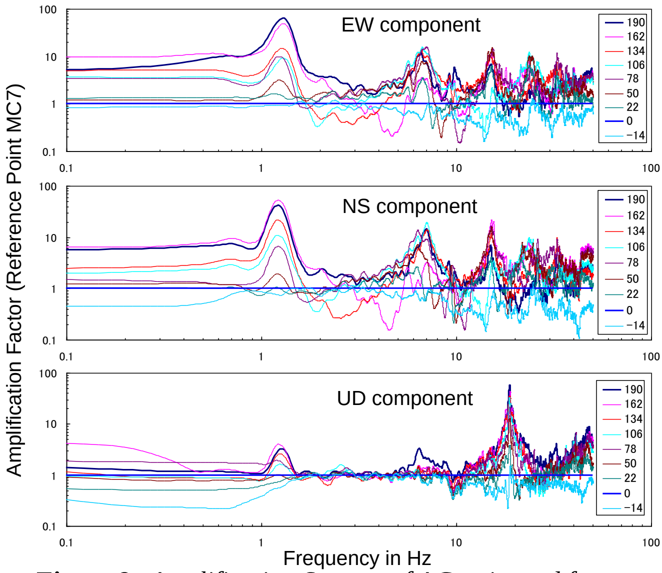


***Figure 8.** Amplification Spectra of AC estimated from Individual Measurement*

(1) Individual Spectral Ratio

Figure 8 shows the individual spectral ratios of each point to point MC7 corresponding to ground surface level. ***Figure 9*** shows the mode shapes of first to third order vibrations. According to this figure, large amplification is observed near the terrace for first order mode same as that of TC, and it seems to reflect the activity of people. The influence for higher mode may be smaller than the result of TC. Anyway, it is necessary for grasping the exact response to conduct measurement at another time. The natural frequencies of longitudinal and torsional vibrations are estimated 18.8 Hz and 9.1 Hz, respectively. From this the propagation velocities are estimated 2300 m/s and 1100 m/s, respectively. The estimated velocity of longitudinal vibration is more than 50 % lower than typical value, 4800 m/s. The shear velocity is also estimated too low to calculate the Poisson's ratio. This is caused by the low frequency of torsional vibration. As seen in ***Photo 1***, there is trace of restoration for vertical damage clearly appears on west side of AC, and this is considered as a reason why the frequency of torsional vibration is very low.

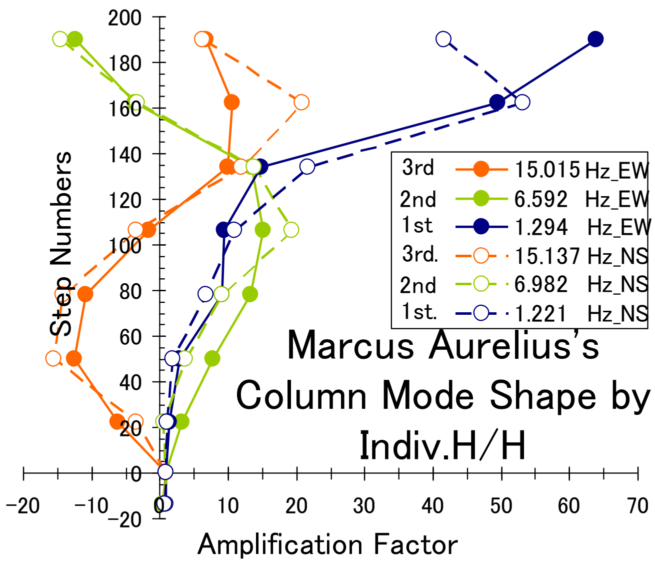


Figure 9. *Mode Shape of AC estimated by Individual Measurements*

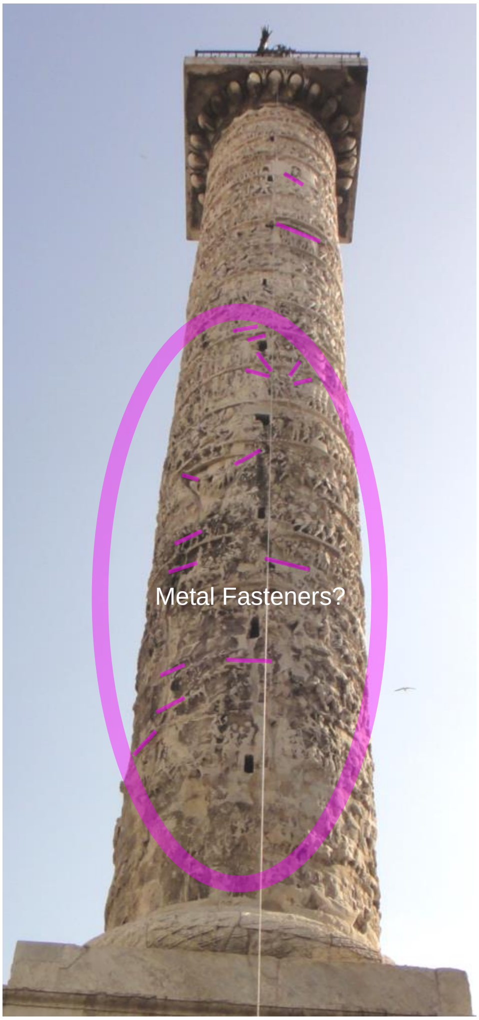


Photo 1. *Damage of AC at West Side*

(2) H/V Spectral Ratio and etc.

Figure 10 shows overlapped H/V spectral ratio of each measurement point. The first order vibration mode shape is considered to similar to the result of the estimation from the H/V based on the discussion for TC. ***Figure 11*** shows the mode shapes of first order vibrations.

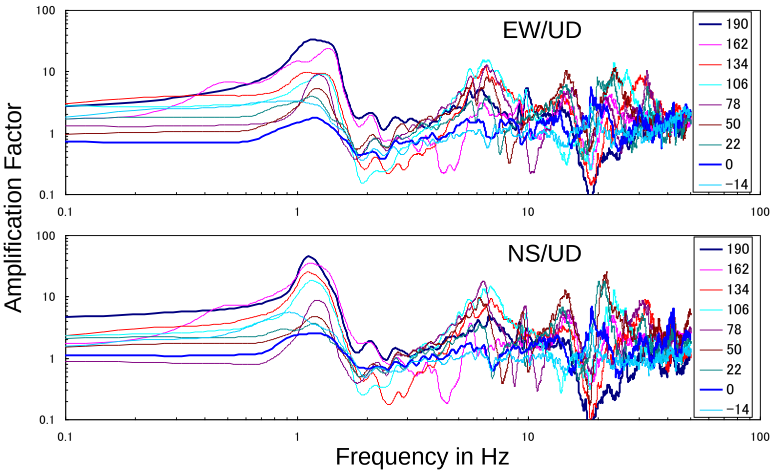


Figure 10. *H/V Spectral Ratio at the sites of AC*

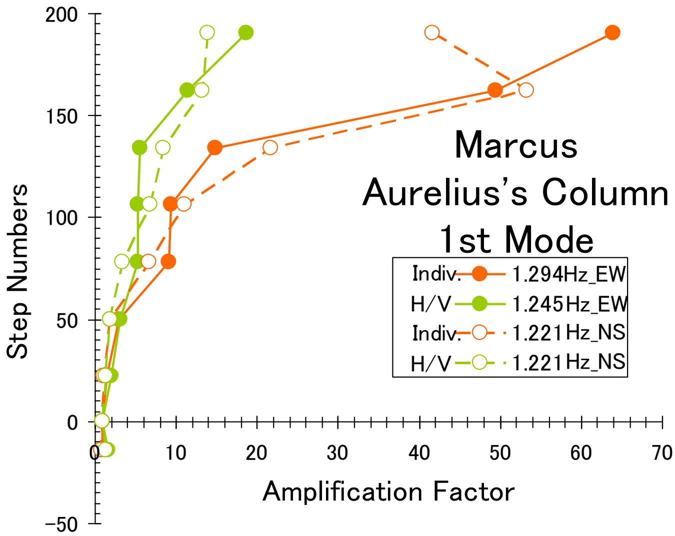


Figure 11. *Comparison between First Mode Shapes of H/V and H/H of AC from Individual Measurements*

The destructive index Kb of AC is calculated from the estimation result of amplification characteristics from H/V ratio, and Kb is compared with the result from the simultaneous measurement of TC. ***Figure 12*** shows the results of comparison of Kb of TC and AC. Kb

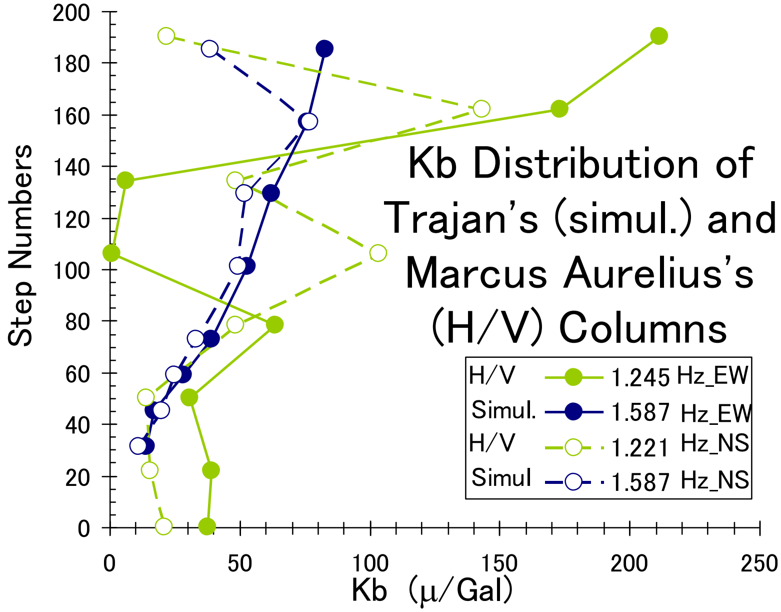


Figure 12. *Comparison between Estimated Kb-values for TC (Blue mark) and AC (Green mark)*

value of TC grows lager towards to higher point reaching maximum 83 μ/Gal. Kb values of AC shows variation at each point, and the maximum value is 212 μ/Gal near the terrace. In other words, drift angle over 1/120 will occur at TC with over 100Gal of seismic acceleration, and in case of AC the same level of drift angle will occur even under 50Gal. These columns are constructed only stacking the large marble blocks with getting stability only by dead load. It is necessary to consider losing the stability of columns by growing the rocking vibration caused by column-soil interaction.

4. Concluding Remarks

This paper describes the results of microtremor measurement for two memorial columns, Trajan's Column and Marcus Aurelius' Column.

Natural frequencies of horizontal vibration are about 1.6 Hz for TC and about 1.3 Hz for AC, and both of them seems to be caused by bending vibration. These are almost similar to previous researches.

The natural frequencies of longitudinal and torsional vibrations of TC are 25.6 Hz and 15.5 Hz, respectively. From this the propagation velocities of the longitudinal vibration and the shear wave are estimated 3100 m/s and 1900 m/s, respectively. These velocities are low value, 65 % of typical value, but the Poisson's ratio is reasonable, 0.33. On the other hand, the natural frequencies of longitudinal and torsional vibration of AC are 18.8 Hz and 9.1 Hz, respectively, and from this the propagation velocities are estimated 2300 m/s and 1100 m/s, and these are very low values. The velocity of longitudinal vibration of AC is extremely low, under 50% of typical value. And the Poisson's ratio could not be calculated because of too slow shear wave velocity. The natural frequency of torsional vibration of AC is extremely low reflecting the longitudinal damage.

For the estimation of vibration mode, the results of simultaneous and individual measurement were compared. Because the memorial columns vibrate easily, it is confirmed that the result of measurement is easily affected by the human activity inside the column and the affection is difficult to avoid without simultaneous measurement. And also it is confirmed that affection can be avoid with extended usage of H/V spectrum ratio.

According to the results of calculating the destructive index Kb of the columns that are based on the fundamental mode of bending vibration, Kb of TC grows larger toward to the top reaching a maximum of about 83μ/Gal near the terrace, and that of AC shows variation at each point and the maximum is 212μ/Gal near the terrace. In other words, drift angle over 1/120 will occur over 100Gal of seismic acceleration for TC, and in case of AC the same level of drift angle will occur even under 50Gal. Because both the columns are built with only stacking of marble block, the columns ensure its behavior of the elastic body only with pre-stress caused by its own weight.

Therefore, if the effect of rocking vibration due to the coupling effect between columns and ground becomes large, it is easy to become unstable. It seems that it will be important to conduct a detailed investigation and determine the limit of the ground motion.

By the additional detailed investigation based on the knowledge obtained by this survey and by the sequential investigation with standardized research method, it is expected that the investigation will contribute to the reasonable maintenance or restoration plan with grasping the degree or location of damage quantitatively.

This time, simultaneous measurement was conducted only for TC. It is desirable to measure TC and AC again completely for confirmation.

Acknowledgments

The measurement of memorial Columns in Rome was conducted under the supporting of Professor Valente and Associate Professor Tallini of L'Aquila University. We would like to express our thanks for their helps.

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